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Overcoming Decoherent Effects from Squeezed Vacuum Reservoir

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Abstract A strategy is investigated to overcome the decoherence for open quantum system by controlling external parameters. The results show that output state may be a pure state in some critical magnetic field parameters which depend on the decay rates so that one may perfectly preserve memory of initial single-qubit states at the finial state under some conditions. The way is applied to two-level atomic systems interacting with squeezed vacuum reservoir.

Keywords Decoherence · Controlling external parameters · Squeezed vacuum reservoir

It is important to understand the interaction between a quantum system with its surrounding environment in both the fundamental theory and application. The decoherence is, on the one hand, a possible explanation of the famous Schrödinger cat paradox [1–4], where a macroscopic superposition is shown how to emerge from a microscopic superposition. For the macroscopic systems, the interaction with environment can never be escaped because the decay rate is proportional to the macroscopic separation between two classical states [5, 6]. Therefore, a linear superposition of macroscopically distinguishable states is immediately changed into a correspondingly statistical mixture, without leaving any space of quantum coherence. The progressive decoherence of a mesoscopic Schrödinger cat was firstly observed in the experiment [7], where the linear superposition of two coherent states under

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an electromagnetic field in a cavity with classically distinct phases has been generated and detected.

On the other hand, the decoherence leads to an irreversible loss of quantum states on the system. The process limits the ability to maintain pure quantum states due to coupling with the environment in quantum information processing [8]. Therefore, it has become a major challenge how to preserve quantum coherent state in the presence of decoherence over the past decades [9–15]. Several schemes have been proposed in the theory of quantum computation and communication, which included quantum error correction strategies [16–19], feedback implementations [20–22], the realization of qubits in symmetric subspaces decoupled from the environment [23–25], dynamical decoupling techniques [26, 27], and engineering of pointer states [28]. It is known that, though the engineering of pointer states and feedback implementations were proposed to maintain a single-qubit state, the engineering approach may lead to new decoherent sources because of the artificial reservoirs and the singularities are a well-known feature of feedback implementations by tracking control [29–32]. Moreover, the others are inadequate to be applied to a single-qubit open system, where both quantum error correcting codes and decoherence-free subspaces are dependent on an encoding of single- into several-qubit states and the dynamical decoupling is inapplicable in a fully decoherent regime.

It is known that the current quantum computation presents a wide range of challenges to quantum information, particularly the search for devices of quantum memory that allow temporal storage of quantum information in a long distance quantum communication [33–36]. Moreover, the application of the geometric quantum computation [37–40] has motivated our studies how to the overcoming the decoherent effects in the real situation.

In this work we propose a simply physical strategy to overcome the decoherence and a given single-qubit state is protected against the destructive effects of interaction between the squeezed vacuum reservoir and two-level atomic system. By controlling some external parameters of physical system to satisfy a special relation with the squeezed parameters, one may obtain an output of quantum pure state. By choosing some outputs for the evolving times, furthermore, we may protect an initial quantum state with a perfect fidelity.

A broadband squeezed vacuum forms a reservoir characterized by a phase-sensitive white noise. The interaction of squeezed radiation with atomic systems results in some unusual properties [41–46]. It is interesting, therefore, in studying how to overcome the decoherent effects from squeezed vacuum reservoir.

Let us consider the coupling of a two-level atom in an electromagnetic field interacting with an incident squeezed light. A master equation can be derived in Schrödinger picture for the atomic density operator [41, 42] under the rotating wave approximation and constant spectrum, such as

$$\begin{aligned}\dot{\rho} = &-\frac{i}{\hbar}[H,\rho] + \frac{1}{2}\gamma(N+1)\{2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\} \\ &+ \frac{1}{2}\gamma N\{2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+\} \\ &- \frac{1}{2}\gamma M\{2\sigma_+\rho\sigma_+ - \sigma_+\sigma_+\rho - \rho\sigma_+\sigma_+\} \\ &- \frac{1}{2}\gamma M^*\{2\sigma_-\rho\sigma_- - \sigma_-\sigma_-\rho - \rho\sigma_-\sigma_-\},\end{aligned} \tag{1}$$

where the first term is an interacting between the magnetic field and two-level atom, the second and third terms are similarly from the normal vacuum reservoir, and the others are from the squeezed vacuum reservoir. While $H = \frac{1}{2}\hbar\omega\sigma_3$, $\omega = g(\mu)B/\hbar$ with $g(\mu)$ that is gyromagnetic and B acts as an external controllable parameter and can be experimentally changed, and $\sigma_3 = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$ and $\sigma_- = |g\rangle\langle e|$ with the relations, such as $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$, of Pauli operators σ_i $(i = 1, 2, 3)$, in which $|g\rangle(|e\rangle)$ is ground (exited) state of the two-level atomic system. And γ is a atomic decay rate for spontaneous emission into a unsqueezed vacuum, N and M are parameters which characterize the squeezing, with $|M|^2 \le N(N+1)$, here the equality holds for a maximally squeezed state. $M = |M|e^{i\phi}$ and the phase ϕ depends on details of the scheme used to generate the squeezed vacuum.

If we set $N = 0$ and $M = 0$ in (1), the atomic density operator is only from the contribution of spontaneous decay of atom in the normal vacuum reservoir. In this case, (1) become the Lindblad's master equation [47]. It is noted that (1) preserves positivity of the density operator $\rho(t)$ for the open system. The solution of (1) is direct. By taking the initial state of two-level atom as $|\psi(0)\rangle = \cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle$, we have

$$\rho_{11} = \cos^2\frac{\theta}{2}e^{-\gamma(2N+1)t} + \frac{N}{2N+1}\left(1 - e^{-\gamma(2N+1)t}\right), \tag{2}$$

and

$$\rho_{22} = \sin^2\frac{\theta}{2}e^{-\gamma(2N+1)t} + \frac{N+1}{2N+1}\left(1 - e^{-\gamma(2N+1)t}\right), \tag{3}$$

which are obvious only from contributions of the second and third terms in (1). Therefore, the diagonal elements (2) and (3) of density matrix are similar to the spontaneous decay of atom in the normal vacuum reservoir. For the nondiagonal elements, without loss of generality, we only consider the case of $\omega_r^2 = \omega^2 - \gamma^2|M|^2 \ge 0$ and find

$$\rho_{12} = \frac{1}{2}\sin\theta\left[\cos(\omega_r t) - \frac{i\omega}{\omega_r}\sin(\omega_r t) - \frac{\gamma|M|}{\omega_r}e^{i\phi}\sin(\omega_r t)\right]e^{-\frac{1}{2}\gamma(2N+1)t}, \tag{4}$$

and

$$\rho_{21} = \frac{1}{2}\sin\theta\left[\cos(\omega_r t) + \frac{i\omega}{\omega_r}\sin(\omega_r t) - \frac{\gamma|M|}{\omega_r}e^{-i\phi}\sin(\omega_r t)\right]e^{-\frac{1}{2}\gamma(2N+1)t}, \tag{5}$$

which are from contributions of all terms in (1). It is noted that ρ_{12} and ρ_{21} are functions of $\cos(\omega_r t)$, $\sin(\omega_r t)$ and an exponential decay factor. So that both the complex oscillations and exponential decay with the evolving time will be included in the nondiagonal elements at the same time. Thus, a similarly optical nutation is emerged in the nondiagonal elements. Here the exponential decay factor $\exp(-\frac{1}{2}\gamma(2N+1)t)$ parameterizes the amount of decoherence. The decoherent effects decrease the size of nondiagonal elements of density matrix in a basis determined by the dephasing interaction with the squeezed environment so that a single-qubit is corrupted by the decoherence.

When the squeezed parameter $|M| = 0$, the oscillations become simple with the frequency ω related to the magnetic field. Therefore, the complex oscillations are one of the squeezed vacuum properties.

The properties of quantum information through noisy quantum channels are usually quantified by the fidelity, which measures the overlap between the initial and time-developed state vectors [48–50]. For an initially pure state $|\psi(0)\rangle$, the fidelity is, in fact, a probability to find the initial state in the output state at a later time. Now we take the density matrix $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$ as an initial state at the time $t = 0$ and $\rho(t)$ in (2)–(5) as a finial state at the time $t = \tau$, respectively. The fidelity of two-level atomic system under the squeezed vacuum environment is written as

$$\begin{aligned} F(\tau) &= \mathrm{Tr}\,(\rho(0)\rho(\tau)) \\ &= \left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right) e^{-\gamma(2N+1)\tau} \\ &\quad + \frac{1}{2N+1}\left(N\cos^2\frac{\theta}{2} + (N+1)\sin^2\frac{\theta}{2}\right)\left(1 - e^{-\gamma(2N+1)\tau}\right) \\ &\quad + \frac{1}{2}\sin^2\theta\left(\cos(\omega_r\tau) - \frac{\gamma M}{\omega_r}\cos\phi\sin(\omega_r\tau)\right) e^{-\gamma(2N+1)\tau}, \end{aligned} \tag{6}$$

which is oscillated in terms of the frequency ω_r as well as an exponential decay in terms of the damping factor $\exp(-\gamma(2N+1)\tau)$. In general, therefore, $0 \le F(\tau) \le 1$. Thus, the part messages are lost in the quantum information processing so that the output state preserves only a part of memory about the initial state. Under the limiting of $F(\tau) = 0$, the initial quantum information is completely lost in the quantum information processing. In the case of $F(\tau) = 1$, the quantum state is perfectly preserved in process of the information.

Our objective is to have the decoherent factor disappear at finial state. Thus we seek the solution of following transcendental equation,

$$\begin{aligned} &-\left(\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}\right)\left(1 - e^{-\gamma(2N+1)\tau}\right) \\ &\quad + \frac{1}{2N+1}\left(N\cos^2\frac{\theta}{2} + (N+1)\sin^2\frac{\theta}{2}\right)\left(1 - e^{-\gamma(2N+1)\tau}\right) \\ &\quad + \frac{1}{2}\sin^2\theta\left(\cos(\omega_r\tau) - \frac{\gamma M}{\omega_r}\cos\phi\sin(\omega_r\tau)\right) e^{-\gamma(2N+1)\tau} \\ &= \frac{1}{2}\sin^2\theta\cos(\Omega_t\tau), \end{aligned} \tag{7}$$

where $\Omega_t = (\omega^2 - \gamma^2|M|^2 - \gamma(2N+1)/4)^{1/2}$. It is worth noting that, as shown at Figs. 1–4, the conditional frequency ω satisfied (7) is dependent on the squeezed parameters and initial angle of single-qubit state, which may be realized by controlling the external parameters. The numeral results at Figs. 1–4 show that there exist, indeed, some physically meaningful solutions in (7) so that the scheme may be implemented to overcome the squeezed decoherence. Inserting (7) into (6), the fidelity becomes

$$F(\tau) = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\Omega_t\tau). \tag{8}$$

From (8), we see that, though the conditional fidelity depends on the squeezed parameters in terms of the conditional frequency Ω_t, the decoherent factor $\exp(-\gamma(2N+1)\tau)$ disappears. It is necessary to emphasize that, in general case, (8) is different from (6) given by (1). However, by controlling the external magnetic field to depend on the squeezed parameters and initial angle of qubit by the experimenters as shown at Figs. 1–4, one may replace (6) by (8) for all evolving time t.

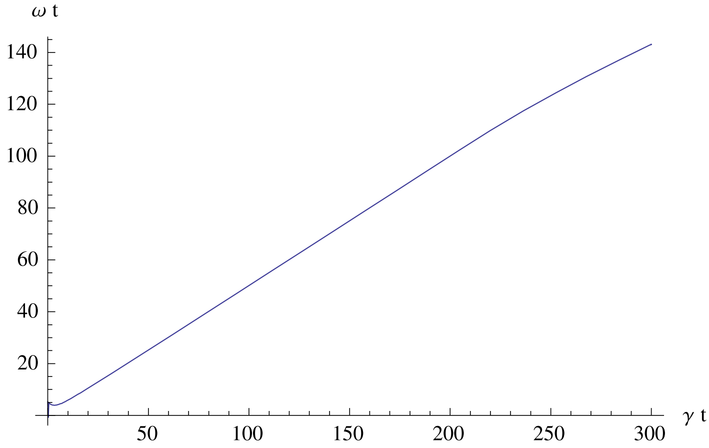

Fig. 1 Numeral solutions of transcendental equation (7) with the parameters $N=0$, $M=0$, $\phi=0$, and $\theta=\pi/2$, which mean that the conditional frequency ω depend not only on squeezed parameters but also on initial azimuthal angle. The results show that one can indeed find the solutions of (7) so that our scheme may be realized effectively to overcome the squeezed decoherence

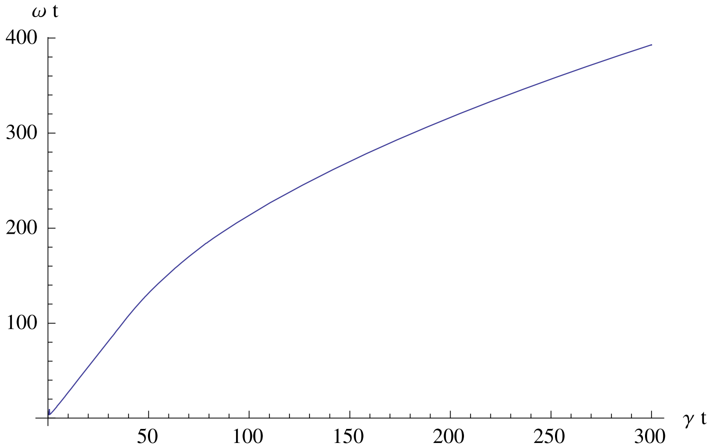

Fig. 2 Same as Fig. 1 for the different parameters with $N=1$, $M=1.0$, $\phi=\pi/4$, and $\theta=\pi/2$, which show that the solutions of (7) are existed for the different parameters

It is obvious that the corresponding output state for (8) may be described by

$$\rho(\tau)=\begin{pmatrix}\cos^2(\theta/2) & \frac{1}{2}\sin\theta e^{-i\Omega_t\tau}\\ \frac{1}{2}\sin\theta e^{i\Omega_t\tau} & \sin^2(\theta/2)\end{pmatrix}. \tag{9}$$

It is well-known that the density matrix of two-level atom can be expressed by a Bloch sphere, such as $\rho=\frac{1}{2}(1+\mathbf{n}\cdot\overrightarrow{\sigma})$, where $\mathbf{n}$ may be parameterized as a Bloch vector along the three pseudospin directions described by two azimuthal angles, such as α and β. Thus,

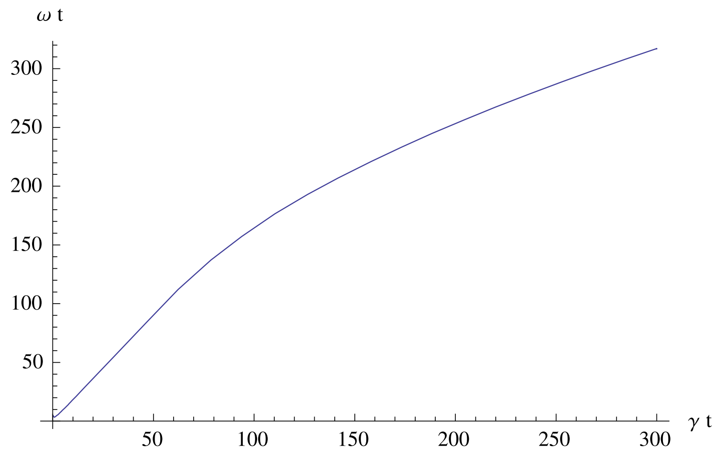

Fig. 3 Same as Figs. 1 and 2 for the different parameters with $N = 2$, $M = 1.0$, $\phi = \pi/3$, and $\theta = \pi/3$

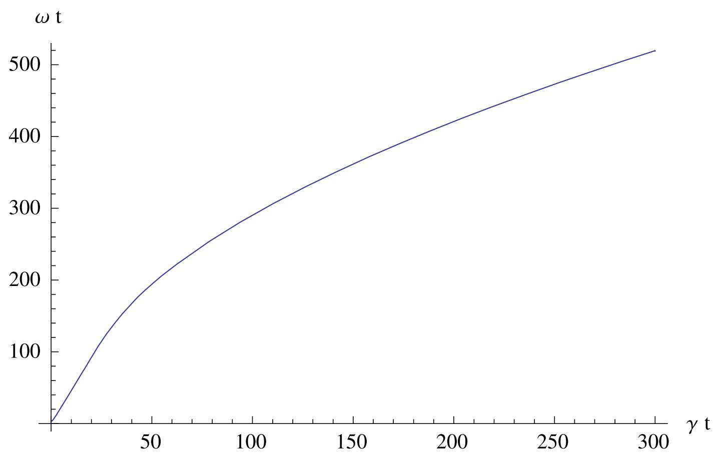

Fig. 4 Same as Figs. 1–3 for the different parameters with $N = 3$, $M = 3.0$, $\phi = \pi/3$, and $\theta = \pi/3$

we have

$$\begin{aligned}\mathbf{n} &= \mathrm{Tr}(\rho\overrightarrow{\sigma}) = (\rho_{12} + \rho_{21}, i(\rho_{12} - \rho_{21}), \rho_{11} - \rho_{22}) \\ &= (r\sin\alpha\cos\beta, r\sin\alpha\sin\beta, r\cos\alpha),\end{aligned} \tag{10}$$

which satisfies the following relations, $\mathbf{n}^* = \mathbf{n}$, and $\mathbf{n}\cdot\mathbf{n} = r^2 \le 1$, where r is radius of the Bloch sphere.

For $r = 1$, the physical system corresponds to a pure state and the given points on the unit Bloch sphere can be mapped onto field amplitudes as a pure state $|\psi\rangle$, where $|\psi\rangle$ is a unit vector in the complex projective Hilbert space. In the case of $r < 1$, the physical system corresponds to a mixed state. In other words, the interior points in the unit Bloch sphere are one-to-one correspondence to mixed states [51–53].

For the physical system described by the density matrix (9), it is easy to find that $r = 1, \alpha = \theta$ and $\beta = \Omega_t \tau$. Thus, the amplitudes of wave functions may be mapped onto given points on the unit Bloch sphere by

$$n_i = \langle \psi | \sigma_i | \psi \rangle, \quad i = 1, 2, 3. \tag{11}$$

By solving (11), a map of one-to-one correspondence for the density matrix (9) is founded by

$$|\psi(\tau)\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\Omega_t \tau} \sin(\theta/2) \end{pmatrix}. \tag{12}$$

It is noted that Ω_t is dependent on the squeezed parameters. In general case, therefore, the state (12) is different from the pure state without interacting with squeezed vacuum reservoir. However, if the output state is controlled at the points of evolving times $\tau = 2n\pi/\Omega_t$ ($n = 1, 2, \ldots$), one may obtain a perfect fidelity $F(\tau) = 1$ according (8). This implies that a single-qubit state is preserved at the finial state in our approach. Thus, an effective scheme is obtained for the recovery of a single-qubit pure state under the condition (7). In other words, by controlling the external magnetic filed according to (7) and Figs. 1–4, one may effectively overcome the decoherence for the output state in the presence of squeezed vacuum reservoir.

In conclusion, an effective strategy is proposed to overcome the decoherence from the squeezed vacuum reservoir, where the controllable magnetic field must be adjusted to depend on the squeezed parameters according (7) (see Figs. 1–4) in advance. Our approach is applied to analyze the two-level system in an external magnetic field interacting with a squeezed vacuum environment. The results show that, by controlling the external magnetic field to satisfy the relation (7) with the squeezed parameters and initial angles, the output state may be a pure state. Therefore, it is very helpful for quantum information processing and coherent controlling.

From (12), we find that the pure states of output may be described by a Hamiltonian $H_R = \frac{1}{2}\Omega_t \hbar \sigma_z$, where the conditional frequencies Ω_t is obtained by the transcendental equation (7) and has a wide choice as shown at Figs. 1–4. Therefore, our approach to avoid the decoherence may be obtained by renormalizing for the free Hamiltonian of two-level system including only the magnetic field, where the magnetic field frequency ω is replaced by the conditional frequencies Ω_t.

Differently from the approaches suppressed the decoherence by the artificial reservoirs, our strategy does not need any such process, which leads to a possible reduction in experimental errors as well as in decoherent sources. In contrast to the feedback of implementations, we may effectively avoid the singularities because of a wide choice for the squeezed parameters and initial angle of single-qubit state. In addition, our approach may be expanded to many-qubit systems.

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